2-1-2008

Modeling Data Envelopment Analysis (DEA) Efficient Location/ Allocation Decisions

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WP # 04 – 03

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ABSTRACT
Many types of facility location/allocation models have been developed to find optimal spatial patterns with respect to location criteria such as: cost, time, coverage, and access. In this paper we develop and test location modeling formulations that utilize aspects of the data envelopment analysis (DEA) efficiency measure to find optimal and efficient facility location/allocation patterns. Solving for the DEA efficiency measure, together with location modeling objectives, provides a promising rich approach to multiobjective location problems.

INTRODUCTION

Many types of facility location/allocation models have been developed to find optimal patterns with respect to different location objectives such as: costs, time, coverage, and access among others. Some of these models have been formulated in a multiobjective programming framework to elicit trade-offs among different and conflicting objectives. In this paper we use the concept of efficiency as defined by data envelopment analysis (DEA) as another objective for location modeling. DEA determines the relative efficiencies of comparable decision making units (DMUs) measured by the ratio of the sum of weighted outputs to the sum of weighted inputs, in which inputs and outputs can be measured in their natural units. Two types of efficiencies can be optimized in this way; spatial efficiency - measured the least cost location and allocation patterns for facilities, and the facility efficiencies in serving demands - measured by the DEA efficiency score for "opened" facilities (those that are chosen to operate in the optimal solution). In the next section, we provide a brief introduction to facility location/allocation and DEA models. Next, we develop and present formulations combining the uncapacitated and capacitated facility location models with the DEA model. Subsequently, we apply these models to a small hypothetical data set and present the results. Finally, the conclusions and future extensions are discussed.

BACKGROUND

Facility Location Models

The classical transportation problem satisfies demands from supply nodes at minimum transportation cost. The uncapacitated facility location problem (UPLP) model extends this by choosing among a number of potential sites for locating supply facilities, those that minimize costs – defined here as the sum of transportation costs and the fixed costs of opening facilities (see Daskin, 1995 pp. 247 – 303, for a comprehensive and cogent review of formulations and solution algorithms for fixed charge location problems) (Balinski 1965). The uncapacitated model assumes each facility has unlimited capacity, and as a result, if a facility supplies a demand node, it will satisfy all the demand, i.e., only one facility is necessary to serve a particular demand. The capacitated facility location problem operates under given supply capabilities/constraints. There have been many extensions to this basic modeling framework (Current and Marsh 1993) that include multiobjective formulations (Current and Ratick, 1996), and dynamic situations (Current, et al. 1998, Osleeb and Ratick 1990, Ratick et al. 1987). In this paper, we will be using the UPLP and the CPLP as the base location modeling framework for our model formulations. The mathematical formulation of the UPLP (I) is:

$$\text{MIN} \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl} dem_l x_{kl} + \sum_{k=1}^{K} F_k y_k \qquad (1)$$

s.t. (I)

$$\sum_{k=1}^{K} x_{kl} = 1 \quad \forall l \qquad (2)$$

$$x_{kl} \le y_k \quad \forall k, l \qquad (3)$$

$$x_{kl}, y_k = 0,1$$

Where:

k = 1, . ., K index of facility locations

l = 1, . ., L index of demand locations

Parameters:

c_{kl} = cost of shipping one unit of demand from facility k to demand l

dem_l = the # of units of demand at l

F_k = fixed cost of opening/using facility k

Decision Variables:

$$x_{kl} = \begin{cases} 1 & \text{if facility k serves demand l} \\ 0 & \text{o/w} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if facility k "opened" (ie. used)} \\ 0 & \text{o/w} \end{cases}$$

The objective function (1) calculates the total cost (transportation and fixed opening costs) of supplying the demand in the system. Total transportation costs are obtained by summing over all facilities and all demands the product of: the per unit transportation cost from facility (k) to a demand (l), the total number of units demanded (dem_l), and the integer variable x_{kl} - which is 1 only if that facility is chosen to supply that demand in the optimal solution. Total fixed costs of opening facilities is obtained by summing over all facilities the product of: the fixed costs (F_k) and the integer variable, y_k, which is 1 if that facility is chosen to be opened in the optimal solution. Because each opened facility provides all the necessary resources to satisfy demands at locations that it serves, the constraints represented in (2) assure that every demand is satisfied, and the constraints in (3) assure that only open facilities can supply demands (and that the fixed cost for opening the facilities are properly assessed in the objective function).

The formulation for the CPLP (II) is given by:

$$\text{MIN}\sum_{k=1}^{K}\sum_{l=1}^{L} c_{kl} b_{kl} + \sum_{k=1}^{K} F_k y_k \qquad (4)$$

s.t. (II)

$$\sum_{k=1}^{K} x_{kl} \geq 1 \quad \forall l \qquad (5)$$

$$x_{kl} \leq y_k \quad \forall k, l \qquad (6)$$

$$\sum_{k=1}^{K} b_{kl} = dem_l \quad \forall l \qquad (7)$$

$$b_{kl} \leq \text{MIN}[dem_l, Cap_k] y_k \quad \forall k, l \qquad (8)$$

$$y_k, x_{kl} = 0,1$$
$$b_{kl} \geq 0$$

Where:

<u>Parameters:</u>

Cap_k = capacity of facility k

b_{kl} = # of units shipped from facility k to demand location l

In CPLP (II), the objective function is similar to UPLP (I) except that the transportation costs is calculated as the product of the per unit transportation costs and the amount shipped from facility k to demand l, i.e., b_{kl}. Constraints in (8) assure that the amount shipped from facility k to demand l, b_{kl}, is either less than the demand requirement at l (dem_l) or the supply at k (Cap_k) - whichever is smaller; if facility k is not opened, then b_{kl} is forced to be zero. The constraints in (5) and (6) are similar to the constraints in (2) and (3), respectfully, in UPLP (I). The requirement that the sum of the amount shipped from all the facilities to each demand location must satisfy the demand location's demand is expressed by the constraints in (7).

Current, Min, and Schilling's (1990) review of 45 facility location papers demonstrated that location models are inherently multiple objective; with the most common objectives classified into four categories: cost minimization, demand-oriented, profit maximization and, environmental concern. The cost minimization objective is the traditional objective of most facility location models. Demand-oriented objectives focus on measuring the "closeness" of the facilities, where "closeness" may be measured in terms of coverage, access, or response time (for a review of covering models see Schilling et. al. (1993) and Toregas et al. (1971)); indicating that there are many attributes that can be used to measure a good location/allocation pattern. In this paper, we suggest that a good location pattern is one that not only optimizes the spatial interaction among facilities, and the demands they serve (as in the above mentioned models), but also optimizes the performance (efficiency) of those facilities at the chosen locations. To evaluate this we add to the spatial interaction measures another objective derived from a technique called data envelopment analysis (DEA), a linear programming approach to measuring relative efficiency among facilities.

Data Envelopment Analysis (DEA) Models

One of the main non-parametric approaches to efficiency measurement is data envelopment analysis (DEA). DEA produces a single aggregate measure of relative efficiency among comparable units (called decision making units (DMUs)) that is a function of the inputs and outputs of processes operating at the DMUs. One advantage of DEA is that these inputs and outputs can remain in their natural physical units without reducing or transforming them into some common measurement such as dollars. DEA defines relative efficiency as the ratio of the sum of weighted outputs to the sum of weighted inputs:

$$\text{DEA Effieicny} = \frac{\text{Sum of weighted outputs}}{\text{Sum of weighted inputs}}$$

The more output produced for a given amount of resources, the more efficient (i.e., less wasteful) is the process. The problem is how each of the individual inputs and output variables, expressed in their different units, are to be weighted. Solving for these weights is the fundamental essence of DEA. For each DMU individually, the DEA procedure finds the set of weights that makes the efficiency of that DMU as large as possible (constrained to be between 0 and 1). The values the derived set of weights can have is restricted by evaluating them in the input/output vectors for all the other comparable DMUs, and constraining those ratios to also be less than or equal to 1. Traditionally, the procedure is then repeated for all other DMUs to obtain their set of weights and associated relative efficiency score. Thus, the DEA solution provides decision makers a listing of comparable DMUs ranked by relative efficiency.

As an example, for the six DMUs shown in Figure 1 below, we will assume that each consumes the same amount of a single input but produces different amounts of outputs y_1 and y_2. The DEA approach finds the convex hull of the non-dominated solutions, an envelope, using mathematical programming. The procedure will identify DMUs P1, P2, P3, and P4 as being efficient; assigning these points, and the locus of points on the line connecting them, an efficiency score of 1. DMUs within the envelope, such as points P5 and P6, will have DEA efficiency scores less than one; in this example their DEA efficiency scores are equal to the ratio of the magnitude of the vector from the origin to the point (say P_5) to the magnitude the vector from the origin through the point to the intersection with the convex hull (P'_5)).

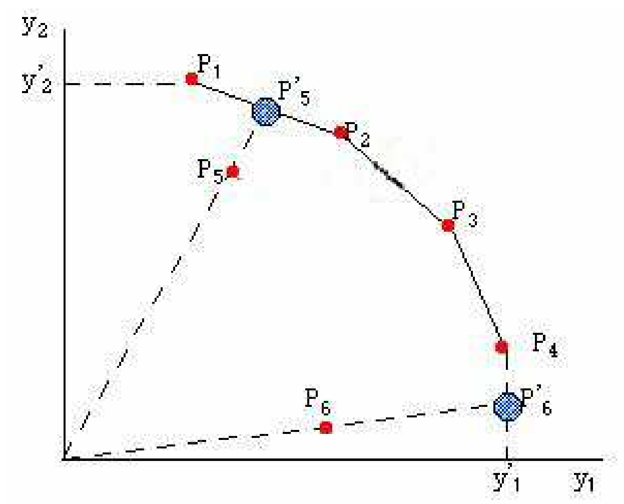

Figure 1. Example of the DEA envelope

Farrell (1957) first described an 'envelope' isoquant approach, in which multiple inputs and one output could be calculated in their natural units, for measuring the efficiency of agricultural production in the United States. Charnes, Cooper and Rhodes (1978) reformulated Farrell's approach and extended this relationship to incorporate multiple outputs as well as multiple inputs. They also provided a methodology for finding the envelope using a fractional linear program, and transformed the fractional program into an easily solved equivalent linear program (described below). The efficiency of the r^{th} decision-making unit (DMU), w_r, can be obtained by solving the following linear fractional CCR DEA formulation (III):

$$\text{MAX } w_r = \frac{\sum_{j=1}^{J} u_j O_{jr}}{\sum_{i=1}^{I} v_i I_{ir}} \qquad (9)$$

s.t. (III)

$$\frac{\sum_{j=1}^{J} u_j O_{jk}}{\sum_{i=1}^{I} v_i I_{ik}} \leq 1 \quad \forall k \qquad (10)$$

$$u_j \geq 0 \; \forall j,$$
$$v_i \geq 0 \; \forall i$$

Where:

$i = 1, . . ., I$ Inputs used at DMU

$j = 1, . . ., J$ Outputs produced at DMU

$k = 1, . ., r, . ., K$ DMUs

<u>Parameters:</u>

O_{jk} = amount of the jth output for the kth DMU

I_{ik} = amount of the ith input for the kth DMU

<u>Decision Variables:</u>

u_j = the weight assigned to the jth output;

v_i = the weight assigned to the ith input.

This formulation determines objectively the set of weights, u_j and v_i that maximize the efficiency, w_r, of DMU r. The constraints in (10) require the ratio of the weighted sum of outputs to the weighted sum of inputs (the efficiency of each DMU, including the rth DMU) not to exceed one using a set of non-negative weights u_j and v_i. A similar DEA formulation is solved sequentially for each DMU. A DMU is considered relatively inefficient ($w_r<1$) if increasing its outputs without increasing inputs, or decreasing its inputs without decreasing outputs, is possible. In this way, a DMU's inefficiency is measured relative to the set of efficient DMUs that define the convex hull.

To solve this as a linear program, the denominator in the objective function, (9), is arbitrarily set equal to 1: $\sum_{i=1}^{I} v_i I_{ir} = 1$. Both sides of the constraints in (10) are then multiplied by the sum of the weighted inputs, yielding the linear equivalent constraint set:

$$\sum_{j=1}^{J} u_j O_{jk} \leq \sum_{i=1}^{I} v_i I_{ik} \quad \forall k.$$

Additionally, a special case called weakly efficient causes the DEA model to be modified in practice. A particular DMU may be weakly efficient if, in the solution to the DEA linear programming model, its DEA efficiency score is 1 and one or more of its weights are equal to zero, but, it is dominated by points on the convex hull (i.e., DMU P'6 in Figure 1). To address this problem the CCR DEA formulation requires each weight to be greater than ε, a small infinitesimal value to assure that weakly efficient DMUs are not classified as efficient. The modified linear CCR DEA formulation therefore becomes (IV):

$$\text{MAX } w_r = \sum_{j=1}^{J} u_j O_{jr} \qquad (11)$$

st. (IV)

$$\sum_{i=1}^{I} v_i I_{ir} = 1 \qquad (12)$$

$$\sum_{j=1}^{J} u_j O_{jk} - \sum_{i=1}^{I} v_i I_{ik} \leq 0 \quad \forall k \qquad (13)$$

$$u_j, v_i \geq \varepsilon \quad \forall j, i$$

Where:

ε = is a small infinitesimal value.

Since the Charnes, et al.'s 1978 paper, there have been thousands of theoretical contributions and practical applications in various fields using DEA, (Seiford 1995). DEA has been applied to many diverse areas such as: health care, military operations, criminal courts, university departments, banks, electric utilities mining operations, manufacturing productivity, and railroad property evaluation (Klimberg (1998), Klimberg and Kern (1992), Seiford (1995) and Seiford and Thrall (1990)), for situations in which input data is stochastic (Desai et al. 2004) and extended to consider multiple objectives (Klimberg (1998), Klimberg and Puddicombe (1999) and Klimberg et al. (2001)). There have also been applications of DEA to evaluating the efficiency of spatial location patterns. Basing their work on the concept of spatial efficiency proposed by Fisher and Rushton (1979), Desai and Storbeck (1990), Desai, Haynes, and Storbeck (1995), and Athanassopoulos and Storbeck (1995), in a series of related papers applied DEA to measure the relative spatial efficiency of location decisions. As part of their DEA models, they used two measures of access as input variables, the total travel distance and the extent of noncoverage (populations not within a specified distance of a facility). Another application of DEA in the context of location/siting is Schroff et al.'s study, (1998) of siting long-term care facilities; in which they describe their problem as one of "locational benchmarking" and used DEA to measure the relative efficiencies of potential geographical regions to determine the location of long-term care facilities.

COMBINED LOCATION/DEA MODELS

The typical solution process for DEA consists of sequentially solving model IV for each DMU. In order to simultaneously consider both patterns of locations for facilities and the associated relative efficiencies of those facilities at each location, the DEA model solution process needs to be modified to allow for the DEA efficiencies of all the DMUs to be calculated in one linear program. In their study of obnoxious-facility location, Thomas et al. (2002), combined a facility location model with DEA by iteratively executing the location/DEA models. An iteration consists of first solving a facility location model and identifying an optimal facility location; then using this optimal facility location as input, a modified DEA model, called a multi-alternative DEA model, is solved simultaneously for all locations. All possible combinations of the facilities must be iteratively tested. Their multi-alternative DEA model examined each of the locations at one time, and their approach did not simultaneously find the efficiencies of all locations. We address this issue in the following formulation.

To modify the CCR DEA linear program for incorporation into the facility location models, we define d_r as the level of inefficiency of DMU r (d_r = 1-w_r). Then, separating out DMU r in constraint (13) and incorporating d_r, we get:

$$\sum_{j=1}^{J} u_j O_{jr} - \sum_{i=1}^{I} v_i I_{ir} + d_r = 0 \qquad (14)$$

Additionally, since constraint (12) applies to DMU r, we can substitute 1 for the weighted sum of outputs in (14), resulting in:

$$\sum_{j=1}^{J} u_j O_{jr} + d_r = 1 \qquad (15)$$

With this adjustment to constraint (13), and additionally extending this modification to the objective function, the modified DEA formulation is now (V):

$$\text{MAX } w_r = 1 - d_r \quad (16)$$

st. (V)

$$\sum_{i=1}^{I} v_i I_{ir} = 1 \quad (17)$$

$$\sum_{j=1}^{J} u_j O_{jr} + d_r = 1 \quad (18)$$

$$\sum_{j=1}^{J} u_j O_{jk} - \sum_{i=1}^{I} v_i I_{ik} \leq 0 \quad \forall k \neq r \quad (19)$$

$$u_j, v_i \geq \varepsilon \quad \forall j, i$$

$$d_r \geq 0 \quad \forall r$$

We extend model V to solve for all DMUs simultaneously as shown below (VI):

$$\text{MAX} \sum_r (1 - d_r) = \sum_r w_r \quad (20)$$

st. (VI)

$$\sum_{i=1}^{I} v_{ri} I_{ir} = 1 \quad \forall r \quad (21)$$

$$\sum_{j=1}^{J} u_{rj} O_{jr} + d_r = 1 \quad \forall r \quad (22)$$

$$\sum_{j=1}^{J} u_{rj} O_{jk} - \sum_{i=1}^{I} v_{ri} I_{ik} \leq 0 \quad \forall k; \forall r; k \neq r \quad (23)$$

$$u_{rj}, v_{ri} \geq \varepsilon \quad \forall j, i, r$$

Where:

Decision Variables:

u_{rj} = the weight assigned to the jth output for DMU r;

v_{ri} = the weight assigned to the ith input for DMU r;

To allow for the simultaneous solution of the DEA model for all DMUs, the objective function

(20), now maximizes the sum of the efficiencies. The constraints in (21) require the sum of DMU r's weighted inputs to be equal to 1, and they are written for each DMU. The constraints in (22) define efficiency as the sum of DMU r's weighted outputs; these constraints are also written for each DMU. The constraints in (23), require the sum of each set of weighted outputs to be less than the corresponding sum of weighted inputs (note that in (23) r sets of k constraints are written because the weights for each r need to be tested with the input/output vectors for all other DMUs).

The combination of model VI above for the simultaneous DEA model, with the UPLP (model I), results in the following formulation (VII) for the fixed simultaneous DEA/UPLP model:

$$\text{MAX} \sum_{k} \sum_{l} (1 - d_{kl}) \qquad (24)$$

$$\text{MIN} \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl}\, dem_l\, x_{kl} + \sum_{k=1}^{K} F_k y_k \qquad (25)$$

(VII)

s.t.

$$\sum_{k=1}^{K} x_{kl} = 1 \quad \forall l \qquad (26)$$

$$x_{kl} \le y_k \quad \forall k, l \qquad (27)$$

$$\sum_{i=1}^{I} v_{kli} I_{ikl} = x_{kl} \quad \forall k, l \qquad (28)$$

$$\sum_{j=1}^{J} u_{klj} O_{jkl} + d_{kl} = x_{kl} \quad \forall k, l \qquad (29)$$

$$\sum_{j=1}^{J} u_{klj} O_{jrs} - \sum_{i=1}^{I} v_{kli} I_{irs} \le 0 \quad \forall k, l; \forall r, s; (k \ne r \text{ and } l \ne s) \qquad (30)$$

$$u_{klj} \ge \varepsilon x_{kl} \quad \forall\, k, l, j \qquad (31)$$

$$v_{kli} \ge \varepsilon x_{kl} \quad \forall\, k, l, i \qquad (32)$$

$$u_{klj} O_{jkl} \le x_{kl} \quad \forall\, k, l, j \qquad (33)$$

$$x_{kl}, y_k = 0,1$$

$$u_{klj}, v_{kli} \ge 0$$

The first objective function, (24), maximizes the sum of the efficiencies for all facility (k), demand (l) combinations opened in the optimal solution; the DMU being evaluated is the facility k/demand l combination being used. A particular facility may have a number of demands that it is chosen to serve in the optimal solution; each of these facility/demand pairs would have an associated DEA efficiency score. The second objective function, (25), and the subsequent two sets of constraints, (26) and (27), are the same as those in model I for the UPLP. The constraints in (28) and (29) are similar to the constraints in (21) and (22) for the simultaneous DEA model, (VI). Additionally, the constraints in (30) are analogous to the constraints in (23) for model VI. When facility k serves demand l, i.e., $x_{kl} = 1$, the corresponding input and output weights are required to be greater than ε on account of constraints (31) and (32), respectively, and further, the constraints in (33) require the weighted outputs to be less than 1, for all facilities, demands, and output types. On the other hand, if facility k does not serve demand l ($x_{kl} = 0$), the constraints in (31) and (32) require the input and output weights to be non-negative, and the constraints in (28) and (33) force them to be equal to 0.

In model VII if a facility k serves demand l, then it will serve all of demand l's requirement. Consequently, there is no interaction between the level of operation and the DEA efficiency. To combine the simultaneous DEA formulation (VI) with the CPLP (II), and to allow for some interaction between location and efficiency, we arbitrarily assumed one of the outputs in the facility's input/output vectors is used to satisfy demands. The amount of that output used in computing the DEA efficiency score for a facility is then a function of whether or not the facility serves a demand, and the activity level of that facility/demand combination. Thus, the DEA efficiencies are dynamically changing as the location and allocation patterns change. Additionally, not all facilities will be compared at optimality, only those that are open. The DEA efficiency score for facilities that are not used is equal to 0. The combination of the simultaneous DEA formulation (VI) with the CPLP (II), results in the following formulation for the adjustable simultaneous DEA/CPLP model (VIII):

$$\text{MAX} \sum_{l} \sum_{k} (1 - d_{kl}) \tag{34}$$

$$\text{MIN} \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl} b_{kl} + \sum_{k=1}^{K} F_k y_k \tag{35}$$

s.t. (VIII)

$$\sum_{k=1}^{K} x_{kl} \geq 1 \quad \forall l \tag{36}$$

$$x_{kl} \leq y_k \quad \forall k,l \tag{37}$$

$$\sum_{k=1}^{K} b_{kl} = dem_l \quad \forall l \tag{38}$$

$$b_{kl} \leq \text{MIN}[dem_l, O_{mkl}] y_k \quad \forall k,l \tag{39}$$

$$\sum_{i=1}^{I} v_{kli} I_{kli} = x_{kl} \quad \forall k,l \tag{40}$$

$$\sum_{j=1}^{J} u_{klj} O_{jkl} + d_{kl} = x_{kl} \quad \forall k,l \tag{41}$$

$$\sum_{j=1}^{J} u_{klj} O_{jrs} - \sum_{i=1}^{I} v_{kli} I_{irs} \leq 0 \quad \forall k,l; \forall r,s; (k \neq r \text{ and } l \neq s) \tag{42}$$

$$u_{klj} \geq \varepsilon x_{kl} \quad \forall k,l,j \tag{43}$$

$$v_{kli} \geq \varepsilon x_{kl} \quad \forall k,l,i \tag{44}$$

$$u_{klj} O_{jkl} \leq x_{kl} \quad \forall k,l \tag{45}$$

$$b_{kl} \geq x_{kl} \quad \forall k,l \tag{46}$$

$$y_k, x_{kl} = 0,1$$

$$b_{kl}, u_{klj}, v_{kli} \geq 0$$

Where:

m = the index of the output that is used to satisfy demand in the CPLP.

The first objective function in the above formulation, (34) and the corresponding DEA-related constraints (40 – 42) are equivalent to objective function (24) and constraints (28 – 30) in the fixed simultaneous DEA/UPLP (VII). The second objective function, (35), and the following four constraints, (36 – 39) are similar to the objective function and constraints in CPLP (II). The only differences are in the constraints in (39), where for the capacity of facility k we use the capacity of one of the outputs, m. The DEA-related constraints (40 – 45) correspond exactly to the DEA-related constraints, (28 – 33), in model VII. The constraints in (46) require at least one unit to be shipped from facility k to demand l, if $x_{kl} = 1$, allowing for that facility to be used in the calculation of DEA efficiency scores. Similar to model VII, if $x_{kl} = 0$, the DEA input and output weights for facility k are equal to 0, and that facility is not considered in the computation of the relative DEA efficiency scores for that location/allocation pattern.

EXAMPLE

An example with seven facilities serving 15 demands, each with four inputs and three outputs, was created and used to test the two DEA/facility location models, models VII and VIII. Figure 2 shows the relative locations of the facilities and the demands; Table 1 lists the demand at each of the 15 demands.

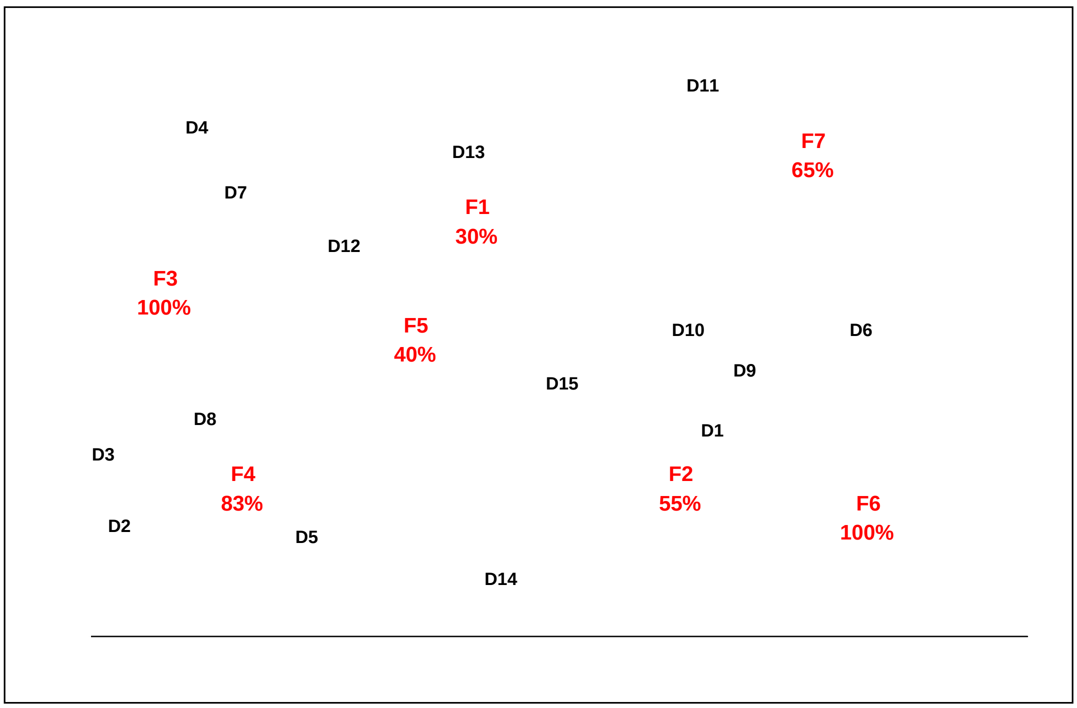

Figure 2. Locations of Facilities (F#) and Demands (D#)

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand	8	28	20	17	23	25	50	49	16	29	45	42	40	8	16

Table 1. Demand requirements at each demand point

Associated with each demand that a facility may serve is a unique input-output vector. Averaging all the DEA scores for all possible demands that can be served by the facility yields an average DEA efficiency score for the facility (the number displayed below each facility in Figure 2). Table A.1 in Appendix A contains the input-output vectors for each of the facility/demand pairs, and their corresponding DEA scores. The remaining parameter values required by models VII and VIII are listed in Appendix A; Table A.2 gives the distances between facilities and demands, and Table A.3 contains the fixed costs for each of the facilities.

The fixed simultaneous DEA/UPLP model, VII, was run with the data discussed above and yielded the following results shown in Table 2.

Table 2. Solution Values for the fixed simultaneous DEA/UPLP problem (VII)

Weights for Uncapacitated Solutions	Min Cost	Max DEA
Relative Weight on Costs*	1	0
Relative Weight on DEA*	0	1
Objective Function Values Total Fixed Costs	$1,950.00	$940.00
Total Transport Costs	$7,786.11	$15,215.76
Total Costs	$9,736.11	$16,155.76
Number of Open Facilities	5	2
Number of facility-demand links	15	15
Total Sum of Efficiency Scores for Open Facilities	10.09	15.00
Average DEA Score for Solution	0.6725	1.0000
Minimum Efficiency Score for Solution	0.2351	1.0000

* - a small weight was used for the objective not being optimized to assure non-dominated multiobjective solutions

Because there is no interaction between the level of operation and the DEA efficiency (which are fixed for this model), we solved for the minimum cost and maximum sum of DEA efficiencies as shown in Table 2. The Min Cost column in Table 2, and visually displayed in Figure 3, shows the solution when the weight on the DEA objective is virtually 0. Five facilities are chosen to serve the demands points with 15 facility-demand links. The average DEA score for all open facility-demand links is a little over 67% and the minimum DEA score is 23.5% (when facility

F1 serves demand 13). As expected all demands are served by their closest facilities, except for demand 15 that should have been served by facility F5, however, the savings in transportation costs do not offset the fixed charge for opening that facility.

The last column in Table 2, labeled the Max DEA solution, and graphically displayed in Figure 4, shows the solution when the weight on costs is virtually 0. In this case, only two facilities are opened with 15 facility-demand links utilized. The average DEA score and the minimum DEA score for all open facility-demand links is 1.000. The fixed costs are significantly lowered from the minimum cost solution, by more than 50%. However, the transportation costs are nearly doubled.

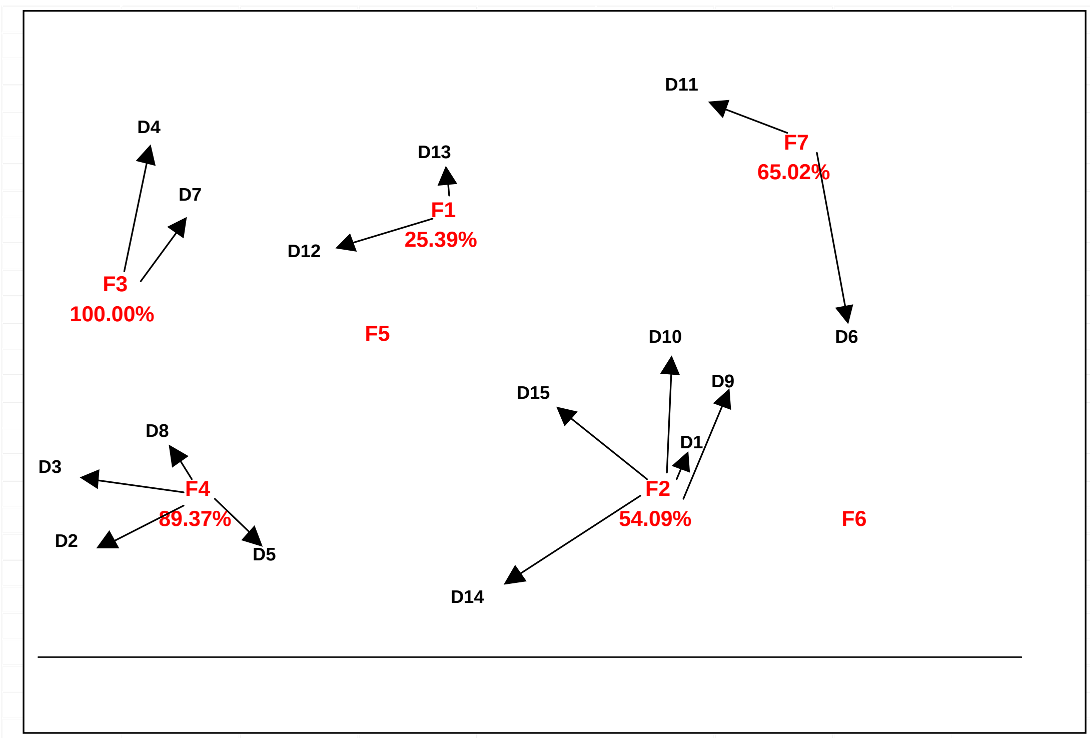

Figure 3. Minimum cost solution to the fixed simultaneous DEA/UPLP location model
Weight on DEA = 0, Weight on Costs = 1

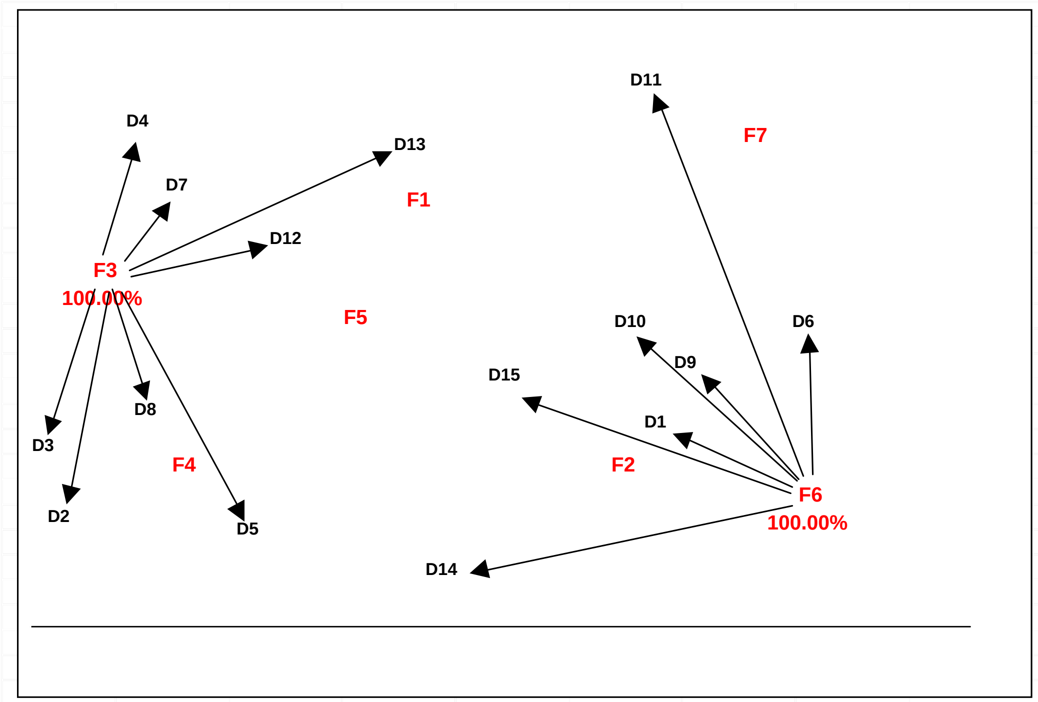

Figure 4. Maximum DEA solution to the fixed simultaneous DEA/UPLP location model Weight on DEA = 1, Weight on Costs = 0

The adjustable simultaneous DEA/CPLP location model formulation, model VIII, was also run with the data presented above and in the Appendix. Here one of the outputs in the DEA vector (Output 3) is the actual commodity transported from the facilities to the demands. The amount of Output 3 in input/output vector for those facility-demand links chosen in the solution will change depending upon the amount of commodity supplied, and concomitantly so will the associated DEA score; increasing output can potentially increase the DEA efficiency score. A series of multiobjective solutions were obtained by varying the relative weights on the DEA objective and the cost objective. Table 3 gives the objective function values for these solutions. The curves in Figure 5 illustrate the tradeoff between costs and DEA efficiency. The lower solid curve shows the tradeoff between total costs and the minimum efficiency score, while the upper dashed curve demonstrates the tradeoff between total costs and the average DEA score. Additionally, three disjointed points are labeled in Figure 5. These points refer to the solutions to the simultaneous DEA/UPLP model. Points A and B apply to the minimum cost solution, where point A is the total cost and minimum efficiency score and point B is the total cost and average DEA score. Lastly, point C corresponds to the maximum DEA solution of the simultaneous DEA/UPLP model.

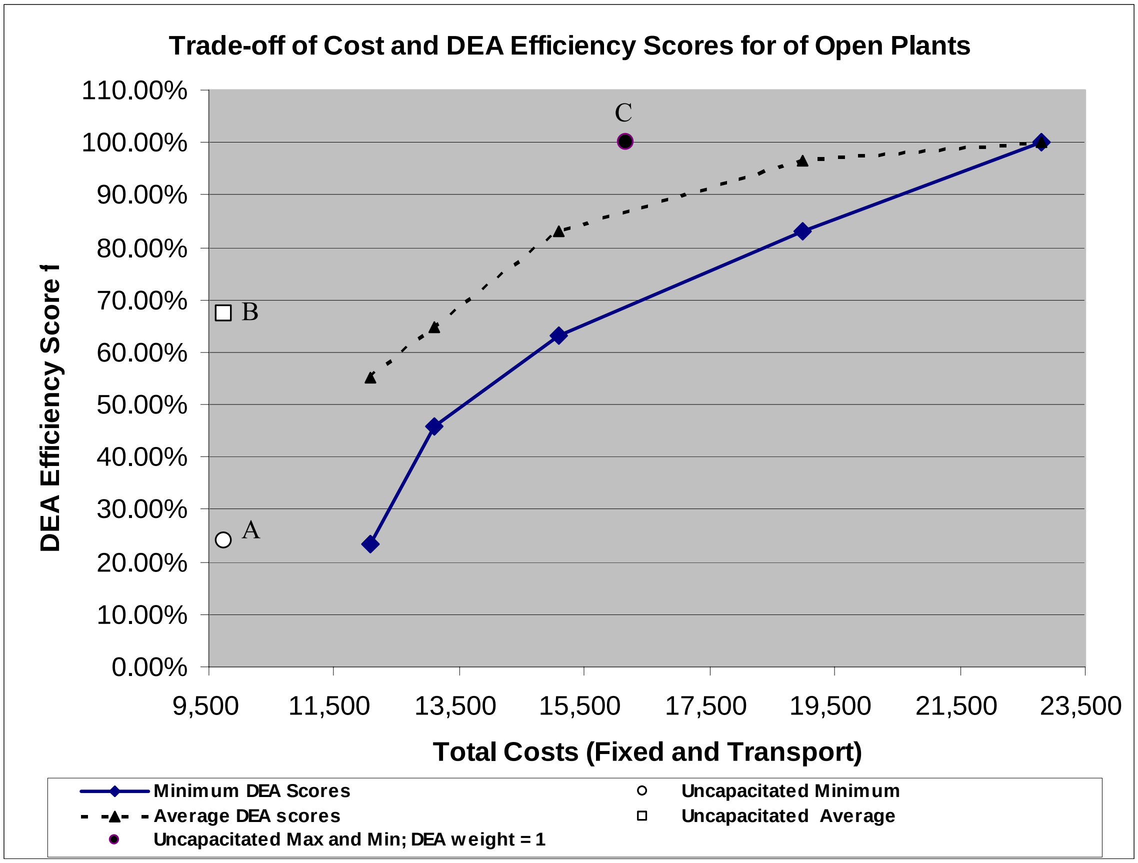

Figure 5. Trade-off of Costs and DEA efficiencies for the Adjustable Simultaneous DEA/CPLP Location Model

Table 3. Solution Values for Adjustable Simultaneous DEA/CPLP location model

Weights for Capacitated Solutions Relative Weight on Costs*	1	0.8	0.7	0.6	0
Relative Weight on DEA*	0	0.2	0.3	0.4	1
Objective Function Values Total Fixed Costs	$3,003	$2,739	$2,248	$2,316	$1,874
Total Transport Costs	$9,066	$10,367	$12,835	$16,662	$20,931
Total Costs	$12,069	$13,106	$15,083	$18,978	$22,805
Number of Open Facilities	6	5	3	3	2
Number of facility-demand links	23	18	21	22	20
Total Sum of Efficiency Scores	12.71	11.69	17.42	21.21	20.00
Average DEA Score for Solution	0.5527	0.6492	0.8296	0.9641	1.0000
Minimum Efficiency Score for Solution	0.2351	0.4586	0.6330	0.8297	1.0000

* - a small weight was used for the objective not being optimized to assure non-dominated solutions. Weights show relative importance in each solution, actual weights were adjusted for the relative sizes of the objective functions.

Figures 6 and 7 graphically show the solutions for DEA weight = 0 and DEA weight = 1, for the adjustable simultaneous DEA/CPLP model formulation (VIII). These two solutions are comparable to the minimum cost and maximum DEA solutions for the fixed simultaneous DEA/UPLP model (VII), Figures 3 and 4. A comparison of the minimum cost solutions shows the fixed simultaneous DEA/UPLP solutions to be less costly; to have a higher average DEA score and to have the same minimum DEA score, Figures 3 and 6. On the other hand, the adjustable simultaneous DEA/CPLP model opened one more facility, F5, and used 23 facility/demand links (while the fixed simultaneous DEA/UPLP model used 15 facility/demand links) because of the restrictions on supply. The average DEA values for each open facility (the average of the DEA efficiency scores for open facility/demand links at a facility) are different for the two models minimum cost solutions (Figures 3 and 6), partially because of the readjustments due to supply constraints, and partially because of the varying output vector for facilities in the adjustable simultaneous DEA/CPLP model (VIII). The two maximum DEA solutions, Figures 4 and 7, are similar except for the split fulfillment of demand caused by supply constraints, leading to costs being lower in the fixed simultaneous DEA/UPLP model, and the adjustable simultaneous DEA/CPLP model using 5 more facility/demand links.

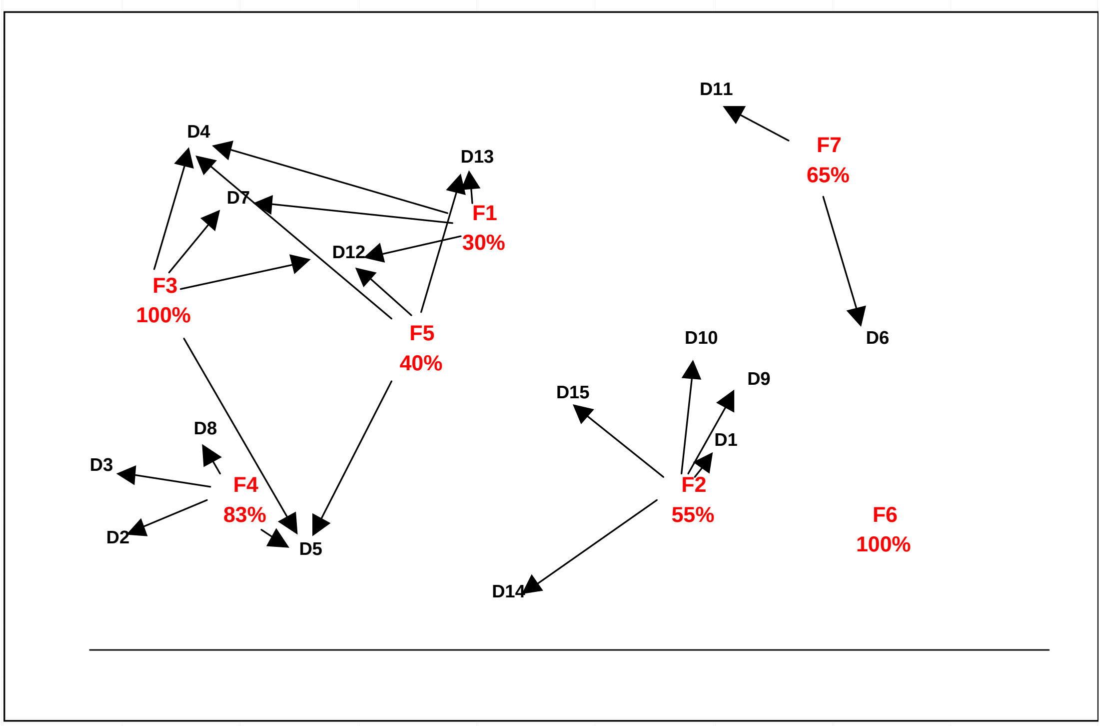

Figure 6. Graphic Solution for the Adjustable Simultaneous DEA/CPLP location model
Weight on DEA = 0, Weight on Costs = 1

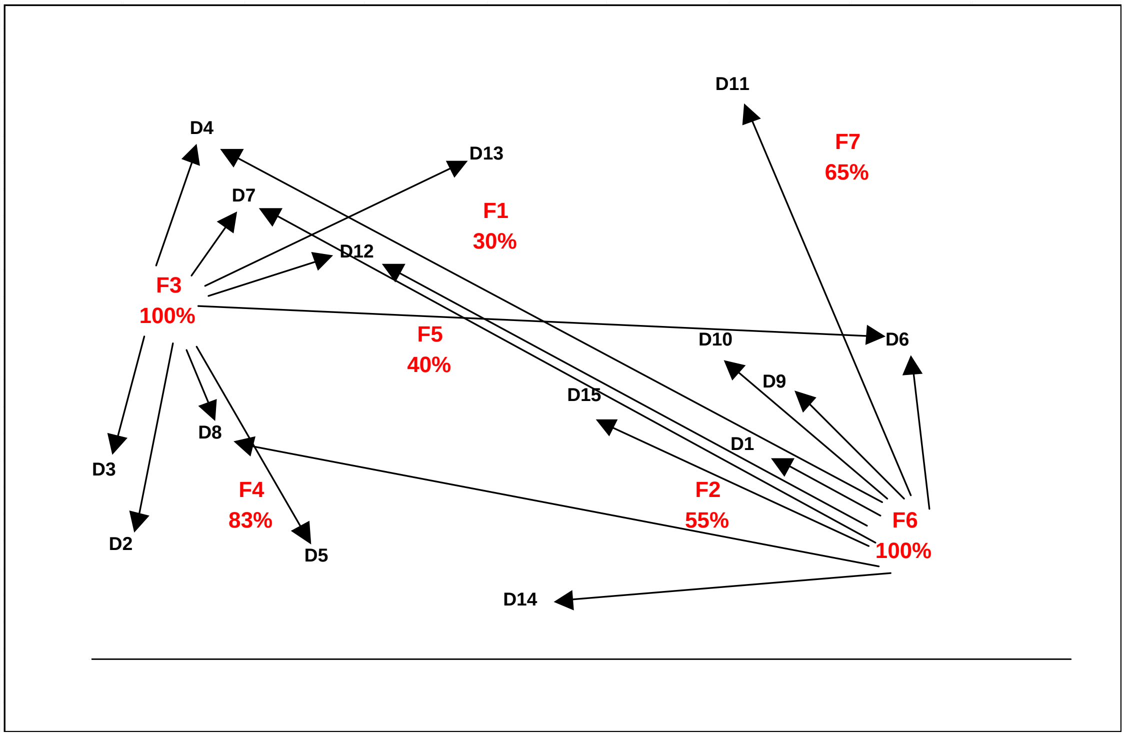

Figure 7. Graphic Solution for the Adjustable Simultaneous DEA/CPLP location model
Weight on DEA = 1, Weight on Costs = 0

CONCLUSIONS AND MODEL EXTENSIONS

The fixed simultaneous DEA/UPLP and adjustable simultaneous DEA CPLP formulations, models VII and VIII, address the issue of solving for the two types of efficiencies described earlier; spatial efficiency - measured by least costs for serving demands; and facility efficiency - measured by the manner in which given inputs are used to produce outputs. We accomplish this by first formulating the simultaneous DEA linear program, and then combining that formulation, in a multiobjective framework, with both the uncapacitated and the capacitated fixed charge facility location problem. The results of those multiobjective formulations demonstrate that this combination may provide promising rich approach to multiobjective location analysis. In model VIII, one of the outputs was used to serve demands, allowing for a more realistic treatment. However, there are two issues with our formulation that require more development and study. The first is to link the production of output to serve demand at a facility to the varying levels of inputs that need to be used to produce those outputs (inputs were exogenous and fixed in the DEA models including model VIII). The other is that, in order to keep the formulation linear, we used the fraction of the total output supplied for the DEA input/output vector. The contribution of that output to the DEA score was not weighted in the DEA solution, but restricted by the fraction of demand shipped. If we had kept the true DEA weight and allowed the amount of output to vary as given in VIII, it would have resulted in a non-linear program. Developing solution schemes for an expanded model VIII, in which outputs, inputs and their corresponding weights are solved for, is another area in which further work needs to be done. While, in this paper, we have used the fixed charge facility location formulation, we assume that similar promising results would obtain if this approach were used with location models formulated with other criteria such as access and coverage; an area we hope to explore in the future. The ultimate test would be to apply the combined DEA/location modeling framework in a real situation.

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Appendix A: Input Data for Example Problems

Table A.1 Input/Output Vectors for Each Facility Serving Each Demand with Associated DEA Scores

Facility	Serving Demand	Amount of Input 1	2	3	4	Amount of Output 1	2	3	DEA Score
F1	1	76	69	63	78	73	4	4	37%
F1	2	50	92	75	98	80	35	30	37%
F1	3	50	76	64	94	69	37	7	36%
F1	4	50	25	24	94	21	36	9	34%
F1	5	79	94	98	86	47	71	85	33%
F1	6	87	90	57	55	52	40	60	33%
F1	7	83	91	23	69	64	32	39	33%
F1	8	26	43	21	84	17	20	35	32%
F1	9	50	98	73	55	14	61	5	32%
F1	10	17	65	24	73	18	10	33	30%
F1	11	75	83	29	54	49	42	15	28%
F1	12	50	74	92	93	56	19	14	27%
F1	13	59	77	68	69	46	5	13	24%
F1	14	70	82	68	32	35	19	6	19%
F1	15	69	69	85	52	16	28	9	14%
F2	1	98	69	41	25	68	34	76	60%
F2	2	29	59	75	59	11	82	9	59%
F2	3	40	39	18	50	46	60	27	59%
F2	4	56	42	90	36	58	82	46	58%
F2	5	61	20	86	83	27	61	96	56%
F2	6	80	48	98	43	95	74	79	56%
F2	7	26	17	34	75	0	46	32	55%
F2	8	64	24	73	69	76	11	26	54%
F2	9	73	56	39	54	76	78	35	54%
F2	10	28	96	21	52	62	18	67	54%
F2	11	55	95	8	42	33	64	38	54%
F2	12	58	79	81	14	28	71	84	53%
F2	13	71	79	51	55	79	89	77	53%
F2	14	18	47	91	74	73	8	65	52%
F2	15	34	84	88	74	99	69	89	52%
F3	1	53	45	25	1	53	95	28	100%
F3	2	1	62	76	32	94	59	38	100%
F3	3	86	2	67	40	61	87	66	100%
F3	4	22	52	41	10	66	63	6	100%
F3	5	18	51	3	67	71	90	81	100%
F3	6	80	17	6	10	85	91	30	100%

Table A.1 Input/Output Vectors for Each FacilityServing Each Demand with Associated DEA Scores (Continued)

Facility	Serving Demand	Amount of Input 1	2	3	4	Amount of Output 1	2	3	DEA Score
F3	7	17	6	59	85	80	46	34	100%
F3	8	21	32	1	27	7	82	16	100%
F3	9	32	8	31	68	49	41	90	100%
F3	10	34	37	51	1	53	58	50	100%
F3	11	21	93	65	15	71	93	88	100%
F3	12	63	5	100	13	51	24	33	100%
F3	13	58	87	68	4	93	1	55	100%
F3	14	2	92	83	79	100	71	82	100%
F3	15	25	18	94	6	10	43	80	99%
F4	1	11	96	65	79	82	36	64	51%
F4	2	56	69	40	66	46	65	76	50%
F4	3	73	72	65	91	18	93	98	50%
F4	4	69	82	49	100	19	98	87	49%
F4	5	45	31	75	72	17	69	12	49%
F4	6	75	51	36	32	69	56	41	48%
F4	7	39	74	96	62	84	57	70	47%
F4	8	39	84	62	98	23	6	92	46%
F4	9	24	83	49	54	48	37	67	46%
F4	10	100	74	39	47	52	97	10	46%
F4	11	76	29	34	98	19	52	53	46%
F4	12	89	16	45	71	56	38	25	46%
F4	13	33	79	86	98	87	24	71	44%
F4	14	38	41	53	7	2	38	3	44%
F4	15	27	62	65	60	64	22	46	43%
F5	1	38	41	53	7	2	38	3	44%
F5	2	27	62	65	60	64	22	46	43%
F5	3	51	26	86	73	52	4	63	42%
F5	4	35	61	1	99	11	13	8	42%
F5	5	37	69	3	58	1	64	8	41%
F5	6	28	28	69	81	41	27	58	40%
F5	7	82	98	42	94	89	60	55	40%
F5	8	99	81	94	42	86	59	41	40%
F5	9	81	83	80	97	93	36	17	40%
F5	10	71	52	82	93	26	82	35	40%
F5	11	64	18	17	22	3	31	21	39%
F5	12	35	95	27	80	15	70	17	38%
F5	13	89	84	37	13	1	59	33	38%
F5	14	52	94	36	39	9	63	24	37%
F5	15	78	70	93	68	34	83	5	37%

Table A.1 Input/Output Vectors for Each FacilityServing Each Demand with Associated DEA Scores (Continued)

Facility	Serving Demand	Amount of Input 1	2	3	4	Amount of Output 1	2	3	DEA Score
F6	1	26	65	73	9	79	37	89	100%
F6	2	1	2	100	78	86	70	97	100%
F6	3	7	50	7	18	73	38	51	100%
F6	4	7	97	39	47	8	81	88	100%
F6	5	72	5	35	56	66	94	64	100%
F6	6	4	84	1	26	87	71	8	100%
F6	7	50	11	88	13	15	89	75	100%
F6	8	10	25	30	49	63	2	83	100%
F6	9	97	5	19	55	7	55	45	100%
F6	10	96	52	52	3	1	58	95	100%
F6	11	18	5	37	59	45	55	83	100%
F6	12	39	62	23	8	16	72	86	100%
F6	13	72	73	6	36	36	11	86	100%
F6	14	97	16	17	74	64	53	95	100%
F6	15	23	21	53	7	60	58	93	100%
F7	1	26	43	44	91	78	56	68	70%
F7	2	22	31	82	66	83	44	19	70%
F7	3	85	29	60	45	95	96	52	69%
F7	4	21	64	97	80	33	90	93	67%
F7	5	31	53	95	49	21	88	31	67%
F7	6	10	67	94	90	72	87	57	67%
F7	7	32	85	79	14	31	4	90	66%
F7	8	30	83	65	69	36	91	99	66%
F7	9	18	27	95	30	21	37	67	64%
F7	10	75	20	67	62	18	26	91	64%
F7	11	95	75	5	56	35	10	57	63%
F7	12	27	88	71	62	43	87	82	63%
F7	13	46	7	93	79	13	49	77	63%
F7	14	80	13	8	39	25	47	11	62%
F7	15	92	3	84	83	53	28	56	61%

Table A.2 Distance Matrix for Facilities to Demands

From Facility	To Demand 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F1	46	69	62	39	57	52	31	48	42	32	37	17	12	60	29
F2	11	72	74	87	49	35	76	62	22	27	68	59	63	27	23
F3	74	39	28	28	45	89	19	22	75	67	77	24	46	64	53
F4	61	17	19	61	11	83	50	13	67	63	90	43	64	36	45
F5	41	49	44	46	36	57	34	30	42	35	57	18	33	41	20
F6	25	96	99	108	72	32	98	87	30	39	76	81	80	48	45
F7	48	108	104	79	91	30	74	90	37	33	18	62	44	81	50

Table A.3 Fixed Costs for Opening Facilities*

Facility	Fixed Cost
F1	250
F2	370
F3	500
F4	330
F5	300
F6	440
F7	430

* This is the fixed cost for opening the facility to serve any of the demands; a smaller fixed cost of 10% of this value is assessed for each demand served.