Fall 2014

MATH 217--Probability and Statistics

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Math 217, Probability and Statistics

Course web page http://aleph0.clarku.edu/~djoyce/ma217
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General description. An introduction to probability theory and mathematical statistics that emphasizes the probabilistic foundations required to understand probability models and statistical methods. Topics covered will include the probability axioms, basic combinatorics, discrete and continuous random variables, probability distributions, mathematical expectation, common families of probability distributions, and the central limit theorem.

Prerequisites. Math 130 Linear Algebra, and Math 131 Multivariate Calculus

Course goals

To provide students with a good understanding of the theory of probability, both discrete and continuous, including some combinatorics, a variety of useful distributions, expectation and variance, analysis of sample statistics, and central limit theorems, as described in the syllabus.

To help students develop the ability to solve problems using probability.

To introduce students to some of the basic methods of statistics and prepare them for further study in statistics.

To develop abstract and critical reasoning by studying logical proofs and the axiomatic method as applied to basic probability.

To make connections between probability and other branches of mathematics, and to see some of the history of probability.

Syllabus

Basic combinatorics. Additive and multiplicative principles, permutations, combinations, binomial coefficients and Pascal's triangle, multinomial coefficients

Kolmogorov's axioms of probability. Events, outcomes, sample spaces, basic properties of probability. Finite uniform probabilities. Philosophies of probability.

Conditional probability. Bayes' formula, independent events, Markov chains

Random variables. Discrete random variables/distributions, expectation, variance, Bernoulli and binomial distributions, geometric distribution, negative binomial distribution, expectation of a sum, cumulative distribution functions

Continuous random variables. Their expectation and variance. Uniform continuous distributions, normal distributions, Poisson processes, exponential distributions; gamma, Weibull, Cauchy, and beta distributions

Joint random variables. Their distributions, independent random variables, and their sums. Conditional distributions both discrete and continuous, order statistics

Expectation. Of sums, sample mean, of various distributions, moments, covariance and correlation, conditional expectation

Limit theorems. Chebyshev’s inequality, law of large numbers, central limit theorem

Textbook. *A First Course in Probability*, 9th edition, by Sheldon Ross. Pearson, 2014.

Course Hours. MWF 9:00–9:50.

Assignments & tests. There will be numerous short homework assignments, mostly from the text, occasional quizzes, two tests during the semester, and a two-hour final exam during finals week.

Time and study. Besides the time for classes, you’ll spend time on reading the text, doing the assignments, and studying of for quizzes and tests. That comes to about five to nine hours outside of class on average per week, the actual amount varying from week to week.

Course grade. The course grade will be determined as follows: 2/9 assignments and quizzes, 2/9 each of the two midterms, and 1/3 for the final exam.

See the course web page for class notes on these topics

Intro to probability via discrete uniform probabilities. Symmetry. Frequency. Simulations and random walks

Background on sets. Unions, intersections, complements, distributivity, DeMorgan's laws. Product, power sets. Countable and uncountable infinities. Combinatorics. Principle of inclusion and exclusion, multiplicative principle, permutations, factorials and Sterling's approximation

Combinations, Pascal's triangle, multinomial coefficients, stars & bars, combinatorial proofs

Axioms for probability distributions.
Probability mass mass functions for discrete distributions, density functions for continuous distributions. Cumulative distribution functions. Sample spaces, axioms, and properties.

Uniform finite probabilities
Odds. Repeated trials. Sampling with replacement. The birthday problem.

Proofs of properties of probability distributions from the axioms.
Conditional probability, definition of conditional probability, the multiplication rule
Assignment 2 due.

Bayes' formula. Examples, tree diagrams

Independent events. Definition, product spaces, independence of more than two events, joint random variables, random samples, i.i.d. random variables
Bertrand's box paradox

The Bernoulli process. Sampling with replacement. Binomial distribution, geometric distribution, negative binomial distribution, hypergeometric distribution. Sampling without replacement

Discrete random variables. Probability mass functions, cumulative distribution functions. Various graphs and charts
Expectation for discrete random variables. Definition, expectation for the binomial and geometric distributions. St. Petersburg paradox.

More on expectation. Properties of expectation.
Variance for discrete random variables. Definition and properties.

Variance of the binomial and geometric distributions.

Continuous probability. Monte Carlo estimates. Introduction to the Poisson process and the normal distribution. Statement of the central limit theorem.

Density functions. Density as the derivative of the c.d.f., and the c.d.f. as the integral of density.

Examples of continuous distributions, functions of random variables, the Cauchy distribution.

The Poisson process. The Poisson, exponential, gamma, and beta distributions. Axioms for the Poisson process.

Expectation and variance for continuous random variables. Definitions and properties. Expectation and variance for uniform and continuous distributions. Lack of expectation and variance for the Cauchy distribution.

The normal distribution, table for the c.d.f. of the standard normal distribution, the normal approximation to the binomial distribution. DeMoivre's 1733 proof for the first instance of the Central Limit Theorem.

Joint distributions. Independent random variables. Joint c.d.f.'s and joint density functions.

Partial derivatives and multiple integrals relating to joint distributions.

Sums and convolution. The discrete case.

More on convolution. The continuous case. Gamma distributions as convolution of exponential distributions. Normal distributions convolute to other normal distributions.

Conditional distributions. Conditional cumulative distribution functions, conditional probability mass functions, conditional probability density functions

Covariance and correlation. Connection of covariance and variance, properties of covariance including bilinearity.
Spurious

Order statistics

Moments and the moment generating function

Joint probability distributions of multivariate functions, the Jacobian.

A proof of the central limit theorem

Discussion of statistical inference.

Bayesian statistics Part I: an example, the basic principle, the Bernoulli process

Maximum likelihood estimators
Discrete case and Continuous case

Bayesian statistics Part II: Bayes pool table, conjugate priors for the Bernoulli process, point estimators, interval estimators

Bayesian statistics Part III: the Poisson process & its conjugate priors

Bayesian statistics Part IV: the normal distribution with known variance

Bayesian statistics Part V: the normal distribution with unknown variance

Common probability distributions
Table of distributions