Fall 2015

MATH 130--Linear Algebra

David Joyce
Clark University, djoyce@clarku.edu

Follow this and additional works at: https://commons.clarku.edu/syllabi

Recommended Citation

Joyce, David, "MATH 130--Linear Algebra" (2015). *Syllabus Share*. 39.
https://commons.clarku.edu/syllabi/39

Math 130 Linear Algebra

Course web page, http://aleph0.clarku.edu/~ma130/
Fall 2015
Professors R. Broker and D. Joyce
Department of Mathematics and Computer Science
Clark University

Course description. Math 130 is a requirement for mathematics and physics majors, and it's highly recommended for majors in other sciences especially including computer-science majors. Topics include systems of linear equations and their solutions, matrices and matrix algebra, inverse matrices; determinants and permutations; real n-dimensional vector spaces, abstract vector spaces and their axioms, linear transformations; inner products (dot products), orthogonality, cross products, and their geometric applications; subspaces, linear independence, bases for vector spaces, dimension, matrix rank; eigenvectors, eigenvalues, matrix diagonalisation. Some applications of linear algebra will be discussed, such as computer graphics, Kirchoff's laws, linear regression (least squares), Fourier series, or differential equations.

Prerequisites. The prerequisite for the course is one year of college calculus, others by permission only.

Assignments & tests. There will be numerous short assignments, mostly from the text, occasional quizzes, two tests during the semester, and a two-hour final exam during finals week in December. The two tests during the semester are yet to be scheduled.

Time and study. Besides the time for classes, you'll spend time on reading the text, doing the assignments, and studying of for quizzes and tests. That comes to about five to nine hours outside of class on average per week, the actual amount varying from week to week. Here's a summary of a typical semester's 180 hours
    Regular class meetings, 14 weeks, 42 hours
    Two evening midterms and final exam, 6 hours
    Reading the text and preparing for class, 4 hours per week, 56 hours
    Doing weekly homework assignments, 56 hours
    Meeting with tutors or in study groups, variable 4 to 12 hours
    Reviewing for midterms and finals, 12 hours

Course grade. The course grade will be determined as follows:
    2/9 assignments and quizzes,
    2/9 each of the two midterms, and

1/3 for the final exam.

Matlab. There are several different symbolic mathematics programs. We'll use the one called Matlab. A couple of others you may have heard of are Maple and Mathematica. They can be used to perform various mathematical computations. You'll need to know how to do these computations and perform small computations by hand, but for large computations, it helps to have a program do them to save time and reduce mistakes.

Course goals

To provide students with a good understanding of the concepts and methods of linear algebra, described in detail in the syllabus.
To help the students develop the ability to solve problems using linear algebra.
To connect linear algebra to other fields both within and without mathematics.
To develop abstract and critical reasoning by studying logical proofs and the axiomatic method as applied to linear algebra.

LEEP learning outcomes

Knowledge of the Natural World and Human Cultures and Societies—including foundational disciplinary knowledge and the ability to employ different ways of knowing the world in its many dimensions. Students will
develop an understanding of linear algebra, a fundamental knowledge area of mathematics,
develop an understanding of applications of linear algebra in mathematics, natural, and social science, and
develop an appreciation of the interaction of linear algebra with other fields

Intellectual and Practical skills—including inquiry and analysis, the generation and evaluation of evidence and argument, critical and creative thinking, written and oral communication, quantitative literacy, and information literacy. Students will
be able to employ the concepts and methods described in the syllabus,
acquire communication and organizational skills, including effective written communication in their weekly assignments, and
be able to follow complex logical arguments and develop modest logical arguments

Personal and Social Responsibility—including ethical reasoning and action, the intercultural understanding and competence to participate in a global society, civic knowledge and engagement locally as well as globally, and the lifelong habits of critical self-reflection and learning. Students will
begin a commitment to life-long learning, recognizing that the fields of mathematics, mathematical modeling and applications advance at a rapid pace, and
learn to manage their own learning and development, including managing time, priorities, and progress

Ability to Integrate Knowledge and Skills—including synthesis and advanced accomplishment across general and specialized studies, bridging disciplinary

and interdisciplinary thinking, and connecting the classroom and the world. Students will

recognize recurring themes and general principles that have broad applications in mathematics beyond the domains in which they are introduced,

understand the fundamental interplay between theory and application in linear algebra, and

be able to solve problems by means of linear algebra

Capacities of Effective Practice—including creativity and imagination, problem solving, self-directedness, resilience and persistence, and the abilities to collaborate with others across differences and to manage complexity and uncertainty. Students will

apply their knowledge toward solving real problems

The text and class discussion will introduce the concepts, methods, applications, and logical arguments; students will practice them and solve problems on daily assignments, and they will be tested on quizzes, midterms, and the final.

Textbook. *Linear Algebra, Concepts and Methods*, by Martin Anthony and Michele Harvey

Syllabus

We won't cover all of the topics listed below at the same depth. Some topics are fundamental and we'll cover them in detail; others indicate further directions of study in linear algebra and we'll treat them as surveys. Besides those topics listed below, we will discuss some applications of linear algebra to other parts of mathematics and statistics and to physical and social sciences.

Matrices and vectors

Matrices. Matrix addition and scalar multiplication. Matrix multiplication. Matrix algebra. Matrix inverses. Powers of a matrix. The transpose and symmetric matrices. Vectors: their addition, subtraction, and multiplication by scalars (i.e. real numbers). Graphical interpretation of these vector operations Developing geometric insight. Inner products and norms in Rn: inner products of vectors (also called dot products), norm of a vector (also called length), unit vectors. Applications of inner products in Rn: lines, planes in R3, and lines and hyperplanes in Rn.

Systems of linear equations

The Chinese method of elimination at http://aleph0.clarku.edu/~djoyce/ma105/simultaneous.html. Systems of linear equations, elementary row operations, and reduced echelon form. Homogeneous systems and null space.

Matrix inversion and determinants
Matrix inverses. Elementary matrices. Introduction to determinants, 2x2 and 3x3 determinants, areas of triangles and parallelograms in the plane, volumes of parallelepipeds, Jacobians Characterizing properties and constructions of determinants, cofactors, diagonal and triangular matrices. More properties of determinants, an algorithm for evaluating determinants, determinants of products, inverses, and transposes, Cramer's rule. Permutations and determinants. Cross products.

Rank, range and linear equations
The rank of a matrix. Rank and systems of linear equations. Range.

Vector spaces
Fields. Vector Spaces, their axiomatic definition. Properties of vector spaces that follow from the axioms. Subspaces of vector spaces. Linear span.

Linear independence, bases and dimension
Linear independence. Linear combinations and basis. Span, and independence. Bases. Coordinates. Dimension. Basis and dimension in Rn.

Linear transformations and change of basis
Linear transformations. Linear transformations and matrices. Some linear transformations of the plane R2 Range and null space. Coordinates. Composition and categories. Change of basis and similarity.

Diagonalisation, eigenvalues and eigenvectors
Eigenvalues, eigenvectors, and eigenspaces. Rotations and complex eigenvalues. Diagonalizable square matrices.

Applications of diagonalisation
Powers of matrices. Systems of difference equations. Linear differential equations.

Inner products and orthogonality
Inner products. Norm and inner products in Cn and abstract inner product spaces. Cauchy's inequality. Orthogonality. Orthogonal matrices. Gram-Schmidt orthonormalisation process

Complex matrices and vector spaces
Complex numbers. Dave's Short Course on Complex Numbers. Complex vector spaces. Complex matrices. Complex inner product spaces. Hermitian conjugates. Unitary diagonalisation and normal matrices. Spectral decomposition.
Class notes, quizzes, tests, homework assignments

There is more information on the course web page
http://aleph0.clarku.edu/~ma130/